

Quiz 18

April 14, 2017

Show all work and circle your final answer.

1. (4 points) Write parametric equations in terms of t representing the curve $y = x^2 - 7$ from $-1 \leq x \leq 2$ (include bounds on t).

$$\boxed{x = t, y = t^2 - 7, -1 \leq t \leq 2}$$

2. (4 points) Write parametric equations for a circle centered at $(1, -3)$ with radius 2 (include bounds on t).

$$\begin{aligned} (x-1)^2 + (y+3)^2 &= 4 & \frac{x-1}{2} &= \cos t \rightarrow & \boxed{x = 2\cos t + 1} \\ \left(\frac{x-1}{2}\right)^2 + \left(\frac{y+3}{2}\right)^2 &= 1 & \frac{y+3}{2} &= \sin t \rightarrow & \boxed{y = 2\sin t - 3} \end{aligned} \quad 0 \leq t \leq 2\pi$$

other possibilities: $\boxed{x = 2\sin t + 1, y = 2\cos t - 3, 0 \leq t \leq 2\pi}$ $\boxed{x = 2\cos 2t + 1, y = 2\sin 2t - 3, 0 \leq t \leq \pi}$

3. (4 points) Find the equation of the tangent line to the curve $x = 2t^2, y = \sqrt{t} + t$ at $t = 4$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{2}t^{-1/2} + 1}{4t}$$

$$\left. \frac{dy}{dx} \right|_{t=4} = \frac{\frac{1}{4} + 1}{16} = \frac{5}{64}$$

$$y(4) = \sqrt{4} + 4 = 6$$

$$x(4) = 2(4)^2 = 32$$

$$\boxed{y - 6 = \frac{5}{64}(x - 32)}$$

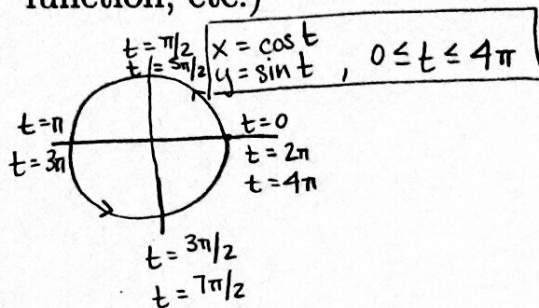
OR

$$\boxed{y = \frac{5}{64}x + \frac{7}{2}}$$

4. (4 points) Write an integral representing the total distance traveled by a particle whose position is given by $x = 3\sin t, y = 4\arctan t$ for $0 \leq t \leq 1$. **Do not evaluate the integral.**

$$L = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \boxed{\int_0^1 \sqrt{(3\cos t)^2 + \left(\frac{4}{1+t^2}\right)^2} dt}$$

5. (4 points) The arclength of a curve C defined by parametric equations $x = f(t)$ and $y = g(t)$, $a \leq t \leq b$, is *NOT* always equal to the distance traveled by a particle with position $(f(t), g(t))$ between $t = a$ and $t = b$. Give an example of when this is not true. (You may draw a picture, explain in words, give a specific function, etc.)



In this case, the particle travels the curve twice between $t=0$ and $t=4\pi$, so the total distance travelled is twice the circumference (arclength) of the circle.